OPGAVE 3.

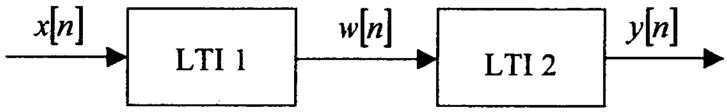

Fig. 1

Fig. 2

Givet to LTI-systemer LTI 1 og LTI 2.

LTI 1 har impulsresponset $h_1[n] = \delta[n] - \delta[n-1]$, og LTI 2 har impulsresponset $h_2[n] = \delta[n] + \delta[n-1]$.

a) Angiv de to systemers differensligninger.

De to systemer kobles i kaskade som vist i fig. 1. I denne konfiguration kan de to systemer erstattes af et enkelt system kaldet LTI 3 som vist i fig. 2.

b) Bestem frekvenskarakteristikken $H(e^{j\hat{\omega}})$ for LTI 3.

c) Vis at $H(e^{j\hat{\omega}})$ kan skrives på formen $2\sin(\hat{\omega}) \cdot e^{j(\frac{\pi}{2}-\hat{\omega})}$.

d) Bestem og skitsér $|H(\hat{\omega})|$.

e) Bestem og skitsér $\arg H(\hat{\omega})$.

LTI 3 påtrykkes et signal: $x[n] = 10 + 5\cos\left(\frac{\pi}{6}n - \frac{\pi}{6}\right)$.

f) Bestem det hertil hørende udgangssignal $y[n]$.

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JHL

a) LTI 1: $h_1[n] = \delta[n] - \delta[n-1]$ dvs: $\underline{\underline{w[n] = x[n] - x[n-1]}}$

LTI 2: $h_2[n] = \delta[n] + \delta[n-1]$ dvs: $\underline{\underline{y[n] = w[n] + w[n-1]}}$

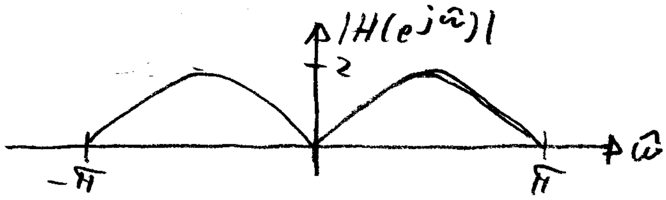
b) Ved identifikation af b_k for de to systemer fås:

$H_1(e^{j\hat{\omega}}) = 1 - e^{-j\hat{\omega}}$ og $H_2(e^{j\hat{\omega}}) = 1 + e^{-j\hat{\omega}}$

$$H(e^{j\hat{\omega}}) = H_1(e^{j\hat{\omega}}) \cdot H_2(e^{j\hat{\omega}}) = (1 - e^{-j\hat{\omega}})(1 + e^{-j\hat{\omega}}) = \underline{\underline{1 - e^{-j2\hat{\omega}}}}$$

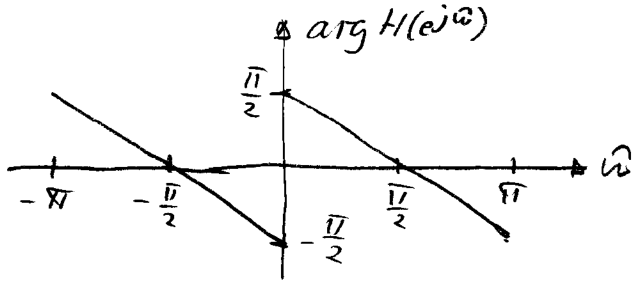
c) $$H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}\left(e^{j\hat{\omega}} - e^{-j\hat{\omega}}\right) = e^{-j\hat{\omega}} \cdot 2j\sin\hat{\omega} = \underline{\underline{2\sin\hat{\omega}\, e^{j(\frac{\pi}{2} - \hat{\omega})}}}$$

d) $|H(e^{j\hat{\omega}})| = 2|\sin\hat{\omega}|$



e) P.g.a. af fortegnsskift for sinus $\hat{\omega}$ fås:

$$\arg H(e^{j\hat{\omega}}) = \begin{cases} \frac{\pi}{2} - \hat{\omega}, & 0 \le \hat{\omega} \le \pi \\ -\frac{\pi}{2} - \hat{\omega}, & -\pi \le \hat{\omega} \le 0 \end{cases}$$



f) $x[n] = 10 + 5\cos\left(\frac{\pi}{6}n - \frac{\pi}{6}\right)$

$H(e^{j0}) = 0$

$H(e^{j\frac{\pi}{6}}) = 2\sin\frac{\pi}{6}\, e^{j(\frac{\pi}{2} - \frac{\pi}{6})} = 1 \cdot e^{j\frac{\pi}{3}}$

$$y[n] = H(e^{j0}) \cdot 10 + |H(e^{j\frac{\pi}{6}})| \cdot 5\cos\left(\frac{\pi}{6}n - \frac{\pi}{6} + \arg H(e^{j\frac{\pi}{6}})\right)$$
$$= \underline{\underline{5\cos\left(\frac{\pi}{6}n + \frac{\pi}{6}\right)}}$$