

Opg 2.7

$$\begin{aligned}
 \text{a)} \quad 3e^{j\frac{\pi}{3}} + 4e^{-j\frac{\pi}{6}} &= 3\left(\cos\frac{\pi}{3} + j\sin\frac{\pi}{3}\right) + 4\left(\cos\frac{\pi}{6} - j\sin\frac{\pi}{6}\right) \\
 &= 3\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) + 4\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right) = \frac{3+4\sqrt{3}}{2} + j\frac{3\sqrt{3}-4}{2} \\
 &= \underline{4.964 + j0.5981} = \underline{5e^{j0.12}}
 \end{aligned}$$

$$\begin{aligned}
 \text{b)} \quad (\sqrt{3} - j3)^{10} &= \left[\sqrt{(\sqrt{3})^2 + (-3)^2} \cdot e^{j\arctan\left(-\frac{3}{\sqrt{3}}\right)} \right]^{10} \\
 &= \left[2\sqrt{3} \cdot e^{-j\frac{\pi}{3}} \right]^{10} = 2^{10} \cdot 3^5 \cdot e^{-j\frac{10\pi}{3}} = 1024 \cdot 243 \cdot e^{j\frac{2\pi}{3}} \\
 &= 248832 \cdot e^{j\frac{2\pi}{3}} = 248832 \cdot \left(\cos\frac{2\pi}{3} + j\sin\frac{2\pi}{3}\right) \\
 &= 248832 \cdot \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \underline{-124416 + j215495}
 \end{aligned}$$

$$\begin{aligned}
 \text{c)} \quad (\sqrt{3} - j3)^{-1} &= \frac{1}{(\sqrt{3} - j3)} = \frac{(\sqrt{3} + j3)}{(\sqrt{3} - j3)(\sqrt{3} + j3)} \\
 &= \frac{\sqrt{3} + j3}{(\sqrt{3})^2 + 3^2} = \frac{\sqrt{3}}{12} + j\frac{3}{12} = \underline{0.1443 + j0.2500} \\
 &= \frac{1}{12} \cdot \sqrt{(\sqrt{3})^2 + 3^2} \cdot e^{j\arctan\left(\frac{3}{\sqrt{3}}\right)} = \frac{2\sqrt{3}}{12} \cdot e^{j\frac{\pi}{3}} \\
 &= \underline{\frac{\sqrt{3}}{6} \cdot e^{j\frac{\pi}{3}} = 0.289 \cdot e^{j1.0472}}
 \end{aligned}$$

- d) I det reelle talsystem betyder \sqrt{a} den positive løsning til ligningen $x^2 = a$, dvs. der er kun én løsning.

I mængden af komplekse tal, er symbolet \sqrt{a} ikke entydigt. Her betyder \sqrt{a} alle rødderne i ligningen $x^2 = a$.

Hvis man i det komplekse system bruger de reelle regler ukritisk, løber man ind i problemer:

$$-1 = j^2 = j \cdot j = \sqrt{-1} \cdot \sqrt{-1} = \sqrt{(-1)(-1)} = \sqrt{1} = 1$$

I dette spørgsmål skal vi altså finde alle løsninger til:

$$z^3 = 2\sqrt{3} \cdot e^{-j\frac{\pi}{3}}$$

hvor $z = re^{j\theta}$ og $0 \leq \theta \leq 2\pi$.

$$z^3 = 2\sqrt{3} \cdot e^{-j(\frac{\pi}{3} + p \cdot 2\pi)}$$

$$\begin{aligned} \Rightarrow z &= (2\sqrt{3})^{\frac{1}{3}} \cdot \left[e^{-j(\frac{\pi}{3} + p \cdot 2\pi)} \right]^{\frac{1}{3}} \\ &= 12^{\frac{1}{6}} \cdot e^{-j(\frac{\pi}{9} + p \cdot \frac{2\pi}{3})} = re^{j\theta} \end{aligned}$$

Vi vælger flg. krav:

$$0 \leq |\theta| \leq 2\pi \quad \Rightarrow \quad p = 0, 1, 2$$

(der selvfølgelig er lige så godt som $0 \leq \theta \leq 2\pi$).

$$\underline{z_1 = 1.513 \cdot e^{-j\frac{\pi}{9}} = 1.422 - j0.5175}$$

$$\underline{z_2 = 1.513 \cdot e^{-j\frac{7\pi}{9}} = -1.159 - j0.9726}$$

$$\underline{z_3 = 1.513 \cdot e^{-j\frac{13\pi}{9}} = -0.2627 + j1.49}$$

- e) $\operatorname{Re}\{je^{-j\frac{\pi}{3}}\} = \operatorname{Re}\{e^{j\frac{\pi}{2}} \cdot e^{-j\frac{\pi}{3}}\} = \operatorname{Re}\{e^{j\frac{\pi}{6}}\} = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} = \underline{0.866}$