Opg 3.1

$$x(t) = A\cos[2\pi(f_c - f_\Delta)t] + B\cos[2\pi(f_c + f_\Delta)t]$$

a) $x(t) = \mathrm{Re}\{Ae^{j2\pi(f_c - f_\Delta)t}\} + \mathrm{Re}\{Be^{j2\pi(f_c + f_\Delta)t}\}$

$= \mathrm{Re}\{Ae^{j2\pi(f_c - f_\Delta)t} + Be^{j2\pi(f_c + f_\Delta)t}\}$

$= \mathrm{Re}\{(Ae^{-j2\pi f_\Delta t} + Be^{j2\pi f_\Delta t})e^{j2\pi f_c t}\}$

$= \mathrm{Re}\{\bar{x}(t)\}$

$\underline{\bar{x}(t) = (Ae^{-j2\pi f_\Delta t} + Be^{j2\pi f_\Delta t})e^{j2\pi f_c t}}$

b) $\bar{x}(t) = (Ae^{-j2\pi f_\Delta t} + Be^{j2\pi f_\Delta t})e^{j2\pi f_c t}$

$= (A\cos 2\pi f_\Delta t - jA\sin 2\pi f_\Delta t)(\cos 2\pi f_c t + j\sin 2\pi f_c t)$
$+ (B\cos 2\pi f_\Delta t + jB\sin 2\pi f_\Delta t)(\cos 2\pi f_c t + j\sin 2\pi f_c t)$

$= A\cos 2\pi f_\Delta t\cos 2\pi f_c t + jA\cos 2\pi f_\Delta t\sin 2\pi f_c t$
$- jA\sin 2\pi f_\Delta t\cos 2\pi f_c t + A\sin 2\pi f_\Delta t\sin 2\pi f_c t$
$+ B\cos 2\pi f_\Delta t\cos 2\pi f_c t + jB\cos 2\pi f_\Delta t\sin 2\pi f_c t$
$+ jB\sin 2\pi f_\Delta t\cos 2\pi f_c t - B\sin 2\pi f_\Delta t\sin 2\pi f_c t$

$\Rightarrow$ $x(t) = \mathrm{Re}\{\bar{x}(t)\} = A\cos 2\pi f_\Delta t\cos 2\pi f_c t + A\sin 2\pi f_\Delta t\sin 2\pi f_c t$
$+ B\cos 2\pi f_\Delta t\cos 2\pi f_c t - B\sin 2\pi f_\Delta t\sin 2\pi f_c t$
$= (A+B)\cos 2\pi f_\Delta t\cos 2\pi f_c t + (A-B)\sin 2\pi f_\Delta t\sin 2\pi f_c t$
$= \underline{C\cos 2\pi f_\Delta t\cos 2\pi f_c t + D\sin 2\pi f_\Delta t\sin 2\pi f_c t}$

med $\underline{C = A+B}$ og $\underline{D = A-B}$

c) Heraf $A = \dfrac{C+D}{2}$ og $B = \dfrac{C-D}{2}$

Ønske: $x(t) = 2\sin 2\pi f_\Delta t\sin 2\pi f_c t$, svarende til $C = 0$ og $D = 2$.

$\Rightarrow$ $A = \dfrac{0+2}{2} = \underline{1}$ og $B = \dfrac{0-2}{2} = \underline{-1}$

Oprindeligt udtryk med $A = 1$ og $B = -1$

$$x(t) = \cos[2\pi(f_c - f_\Delta)t] - \cos[2\pi(f_c + f_\Delta)t]$$

$$= \frac{e^{j2\pi(f_c - f_\Delta)t} + e^{-j2\pi(f_c - f_\Delta)t}}{2} - \frac{e^{j2\pi(f_c + f_\Delta)t} + e^{-j2\pi(f_c + f_\Delta)t}}{2}$$

$$x(t) = \frac{1}{2}e^{j2\pi(f_c - f_\Delta)t} + \frac{1}{2}e^{-j2\pi(f_c - f_\Delta)t} + \frac{1}{2}e^{j\pi}e^{j2\pi(f_c + f_\Delta)t} + \frac{1}{2}e^{j\pi}e^{-j2\pi(f_c + f_\Delta)t}$$

Spektrum:

$$\underline{\{(X, f)\} = \left\{(\frac{1}{2}e^{j\pi}, f_c + f_\Delta), (\frac{1}{2}, f_c - f_\Delta), (\frac{1}{2}, -(f_c - f_\Delta)), (\frac{1}{2}e^{j\pi}, -(f_c + f_\Delta))\right\}}$$