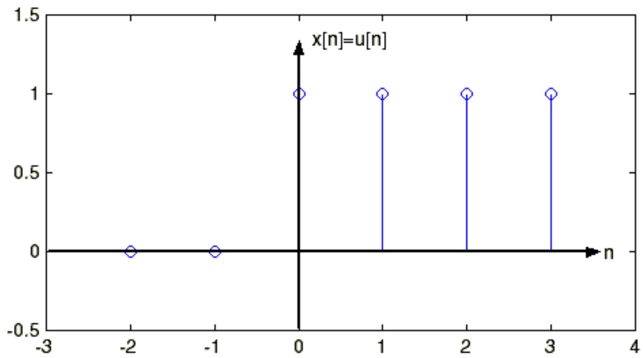
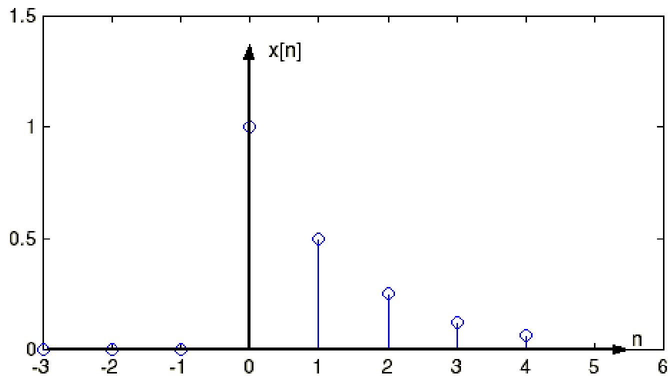
Opg 5.5

a) $x[n] = u[n] = \begin{cases} 0 & \text{for} \quad n < 0 \\ 1 & \quad\quad\; n \geq 0 \end{cases}$



b) $x[n] = (0.5)^n u[n]$

n	0	1	2	3	4
$(1/2)^n$	1	$1/2$	$1/4$	$1/8$	$1/16$



c) $y[n] = \frac{1}{L} \sum_{k=0}^{L-1} x[n-k]$ running average/glidende middelværdi

Antag: $L = 4$, dvs. $M = 3$.

$y[n] = \frac{1}{4} \sum_{k=0}^{3} x[n-k]$ $-5 \leq n \leq 10$

n	-5	-4	-3	-2	-1	0	1	2	3	4
x[n]	0	0	0	0	0	1	1/2	1/4	1/8	1/16
y[n]	0	0	0	0	0	1/4	3/8	7/16	15/32	15/64

n	5	6	7	8	9	10
x[n]	1/32	1/64	1/128	1/256	1/512	1/1024
y[n]	15/128	15/256	15/512	15/1024	15/2048	15/4096

d) Summen af en kvotientrække:

$$\text{sum} = \frac{a(1-q^n)}{1-q} \qquad (a = \text{første led},\ n = \text{antal led},\ q = \text{kvotient})$$

derfor bliver:

$$\sum_{k=M}^{N} \alpha^k = \frac{\alpha^M(1-\alpha^{N-M+1})}{1-\alpha} = \frac{\alpha^M - \alpha^{N+1}}{1-\alpha} \text{ og}$$

$$y[n] = \frac{1}{L}\sum_{k=0}^{L-1} x[n-k] = \frac{1}{L}\sum_{k=0}^{L-1} a^{n-k}u[n-k] = \frac{1}{L}\sum_{l=n}^{n-l=L-1} a^l u[l] \qquad \begin{matrix} l = n-k \\ k = n-l \end{matrix}$$

$$= \frac{1}{L}\sum_{l=n}^{l=n-L+1} a^l u[l] = \frac{1}{L}\sum_{l=n-L+1}^{n} a^l u[l]$$

Alle $u[l]$ i summen er 1, når n bliver stor nok. Kravet er, at nedre grænse
$l = n - L + 1 \geq 0 \quad \Rightarrow \quad n \geq L - 1$.

I så fald haves:

$$y[n] = \frac{1}{L}\sum_{l=n-L+1}^{n} a^l u[l] = \frac{1}{L}\frac{a^{n-L+1} - a^{n+1}}{1-a} = \frac{1}{L}\frac{a^{n+1}(a^{-L}-1)}{1-a}$$

For $n < L-1$ bliver ikke alle $u[l]$ i summen lig 1. For $l < 0$ bliver $u[l] = 0$, og for $l \geq 0$ bliver $u[l] = 1$.

I så fald haves:

$$y[n] = \frac{1}{L}\sum_{l=n-L+1}^{n} a^l u[l] = \frac{1}{L}\sum_{l=0}^{n} a^l = \frac{1}{L}\frac{a^0 - a^{n+1}}{1-a}$$

$$y[n] = \begin{cases} \dfrac{a^{n+1}(a^{-L}-1)}{L(1-a)} & \text{for} \quad n \geq L-1 \\ \dfrac{1-a^{n+1}}{L(1-a)} & \phantom{\text{for}} \quad n < L-1 \end{cases}$$