Opg 6.11

a) LTI: $y[n] = x[n] - 3x[n-1] + 3x[n-2] - x[n-3]$

$\{b_k\} = \{1, -3, 3, -1\} \qquad L = 4,\ M = 3$

$\Rightarrow \quad H(\hat{\omega}) = 1e^{-j\hat{\omega}0} - 3e^{-j\hat{\omega}1} + 3e^{-j\hat{\omega}2} - 1e^{-j\hat{\omega}3} = (1 - e^{-j\hat{\omega}})^3$

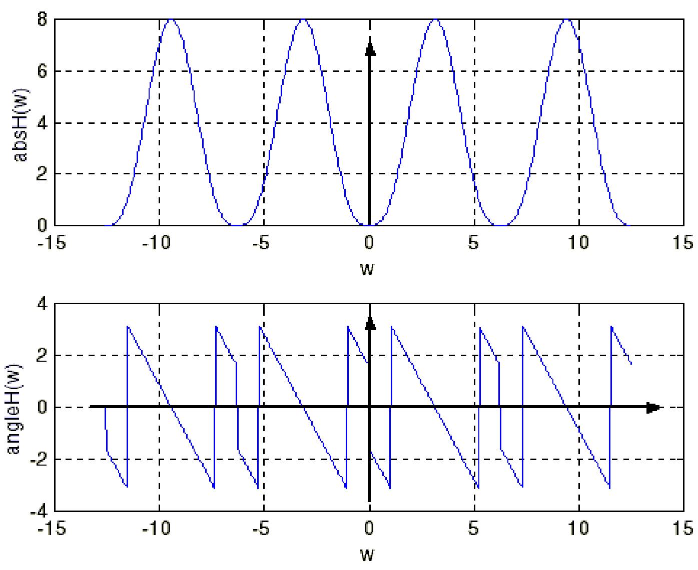
idet $(1-a)^3 = 1 - 3a + 3a^2 - a^3$

$$H(\hat{\omega}) = (1 - e^{-j\hat{\omega}})^3 = \left[e^{-j\frac{\hat{\omega}}{2}}\left(e^{j\frac{\hat{\omega}}{2}} - e^{-j\frac{\hat{\omega}}{2}}\right)\right]^3 = \left[2je^{-j\frac{\hat{\omega}}{2}}\sin\left(\frac{\hat{\omega}}{2}\right)\right]^3$$

$$= \left[2\sin\left(\frac{\hat{\omega}}{2}\right)e^{j\left(\frac{\pi}{2}-\frac{\hat{\omega}}{2}\right)}\right]^3 = 8\sin^3\left(\frac{\hat{\omega}}{2}\right)e^{j\left(\frac{3\pi}{2}-\frac{3\hat{\omega}}{2}\right)} = \underline{8\sin^3\left(\frac{\hat{\omega}}{2}\right)e^{j\left(-\frac{\pi}{2}-\frac{3\hat{\omega}}{2}\right)}}$$

q.e.d.

b)



Mathlab-kode, der genererer figurerne:

```
» w=-4*pi:0.05:4*pi;
» B=[1 -3 3 -1];
» A=[1];
» H=freqz(B,A,w);
» subplot(2,1,1), plot(w,abs(H))
» subplot(2,1,2), plot(w,angle(H))
»
```

$$\arg H(\hat{\omega}) = \begin{cases} -\frac{\pi}{2} - \frac{3\hat{\omega}}{2} & 0 < \hat{\omega} < \frac{\pi}{3} \\ \frac{3\pi}{2} - \frac{3\hat{\omega}}{2} & \frac{\pi}{3} < \hat{\omega} < \frac{5\pi}{3} \\ \frac{7\pi}{2} - \frac{3\hat{\omega}}{2} & \frac{5\pi}{3} < \hat{\omega} < 2\pi \\ \frac{5\pi}{2} - \frac{3\hat{\omega}}{2} & 2\pi < \hat{\omega} < \frac{7\pi}{3} \\ \frac{9\pi}{2} - \frac{3\hat{\omega}}{2} & \frac{7\pi}{3} < \hat{\omega} < \frac{11\pi}{3} \\ \frac{13\pi}{2} - \frac{3\hat{\omega}}{2} & \frac{11\pi}{3} < \hat{\omega} < 4\pi \end{cases}$$

Det er vinklens hovedværdi, der her er beregnet.
Bemærk skiftene på $+2\pi$, $+2\pi$, $-\pi$, $+2\pi$ og $+2\pi$.
Skiftet på $-\pi$ skyldes fortegnsskift for $\sin^3\left(\frac{\hat{\omega}}{2}\right)$.

c) Input: $x[n] = 10 + 4\cos(0.5\pi n + \frac{\pi}{4})$. Bestem output $y[n]$.

$$H(0) = 8\sin^3\left(\frac{0}{2}\right)e^{-j\frac{\pi}{2}} = 0$$

$$H(\frac{\pi}{2}) = 8\sin^3\left(\frac{\pi/2}{2}\right)e^{j\left(-\frac{\pi}{2}-\frac{3\pi}{2}\right)} = 8\left(\frac{\sqrt{3}}{2}\right)^3 e^{-j\frac{5\pi}{4}} = 2.83e^{j\frac{3\pi}{4}}$$

$$y[n] = 0\cdot 10 + 8\left(\frac{\sqrt{3}}{2}\right)^3 \cdot 4\cos(\frac{\pi}{2}n + \frac{\pi}{4} - \frac{5\pi}{4}) = \underline{11.31\cos(\frac{\pi}{2}n - \pi)}$$

$$\underline{= 11.31\cos(\frac{\pi}{2}n + \pi)}$$

d) Input: $x[n] = \delta[n] \quad \Rightarrow \quad y[n] = h[n]$ impulsresponse

Output: $\underline{h[n] = \delta[n] - 3\delta[n-1] + 3\delta[n-2] - \delta[n-3]}$

e) $\underline{y[n] = 11.31\cos(\frac{\pi}{2}n + \pi) + 5\delta[n-3] - 15\delta[n-4] + 15\delta[n-5] - 5\delta[n-6]}$