Opg 6.13

Givet LTI-system ved differensligningen:

$$y[n]=\frac{1}{4}\big[x[n]+x[n-1]+x[n-2]+x[n-3]\big]=\frac{1}{4}\sum_{k=0}^{3}x[n-k]$$

a) $$h[n]=\frac{1}{4}\big[\delta[n]+\delta[n-1]+\delta[n-2]+\delta[n-3]\big]=\underline{\frac{1}{4}\sum_{k=0}^{3}\delta[n-k]}$$

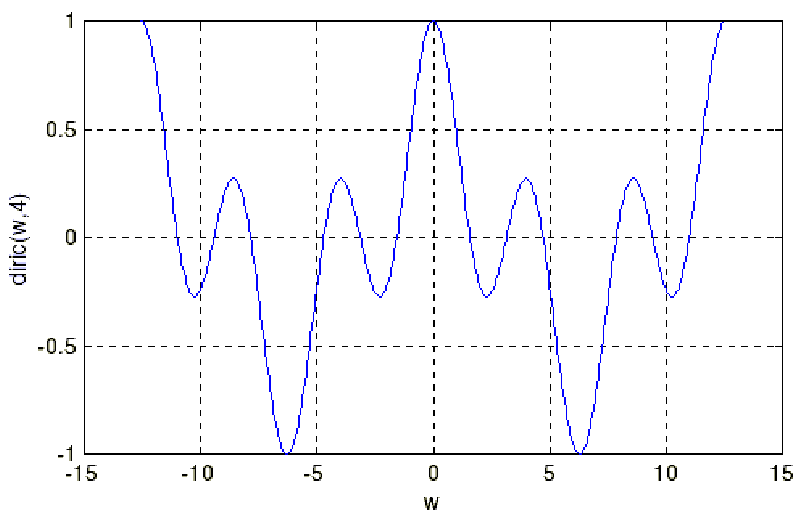
b) $$H(\hat{\omega})=\underline{\frac{1}{4}\left[1+e^{-j\hat{\omega}}+e^{-j2\hat{\omega}}+e^{-j3\hat{\omega}}\right]}$$ kvotientrække

$$=\frac{1}{4}\cdot\frac{1-e^{-j4\hat{\omega}}}{1-e^{-j\hat{\omega}}}=\frac{1}{4}\cdot\frac{e^{-j2\hat{\omega}}\left(e^{j2\hat{\omega}}-e^{-j2\hat{\omega}}\right)}{e^{-j\hat{\omega}/2}\left(e^{j\hat{\omega}/2}-e^{-j\hat{\omega}/2}\right)}$$

$$=\frac{1}{4}\cdot\frac{\sin(2\hat{\omega})}{\sin(\hat{\omega}/2)}\cdot e^{-j3\hat{\omega}/2}=\frac{\sin(4\hat{\omega}/2)}{4\sin(\hat{\omega}/2)}\cdot e^{-j3\hat{\omega}/2}$$

$$=\underline{D_4(\hat{\omega})\cdot e^{-j1.5\hat{\omega}}}$$

c) Den numeriske værdi bliver: $|H(\hat{\omega})|=|D_4(\hat{\omega})|$, hvor $D_4(\hat{\omega})$ er Dirichlets funktion.



Dirichlets funktion.

$$D_L(\hat{\omega})=\frac{\sin(L\hat{\omega}/2)}{L\sin(\hat{\omega}/2)} \qquad L=4$$

Mathlab-plotting:

```
» w=-4*pi:0.05:4*pi;
» d=diric(w,4);
» plot(w,d)
»
```

$$D_4(\hat{\omega}) = \frac{\sin(4\hat{\omega}/2)}{4\sin(\hat{\omega}/2)} = 0 \quad \Leftrightarrow \quad \sin(2\hat{\omega}) = 0 \quad \wedge \quad \sin(\hat{\omega}/2) \neq 0$$

$$\Leftrightarrow \quad 2\hat{\omega} = p\pi \quad \wedge \quad \hat{\omega}/2 \neq q\pi$$

$$\hat{\omega} = p\frac{\pi}{2} \quad \text{hvor} \quad p \in Z \setminus \{0, \pm 4, \pm 8, \pm 12, ...\}$$

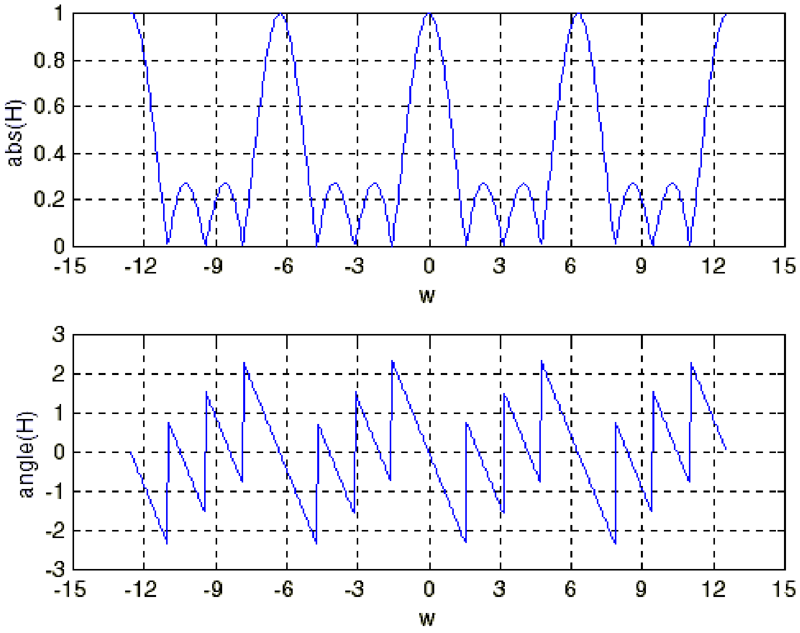
Faseforløbet bliver:

$$\arg H(\hat{\omega}) = \begin{cases} -\frac{3}{2}\hat{\omega} & 0 < \hat{\omega} < \frac{\pi}{2} \\ -\frac{3}{2}\hat{\omega} + \pi & \frac{\pi}{2} < \hat{\omega} < \pi \\ -\frac{3}{2}\hat{\omega} + 2\pi & \pi < \hat{\omega} < \frac{3\pi}{2} \\ -\frac{3}{2}\hat{\omega} + 3\pi & \frac{3\pi}{2} < \hat{\omega} < \frac{5\pi}{2} \\ -\frac{3}{2}\hat{\omega} + 4\pi & \frac{5\pi}{2} < \hat{\omega} < 3\pi \\ -\frac{3}{2}\hat{\omega} + 5\pi & 3\pi < \hat{\omega} < \frac{7\pi}{2} \\ -\frac{3}{2}\hat{\omega} + 6\pi & \frac{7\pi}{2} < \hat{\omega} < 4\pi \end{cases}$$

En nemmere måde at plotte på, er ved hjælp af funktionen freqz:

Mathlab-plotting:

```
» w=-4*pi:0.05:4*pi;
» B=[0.25 0.25 0.25 0.25];
» A=[1];
» H=freqz(B,A,w);
» subplot(2,1,1), plot(w,abs(H))
» subplot(2,1,2), plot(w,angle(H))
»
```



d) $x[n] = 5 + 4\cos(0.2\pi n) + 3\cos(0.5\pi n + \frac{\pi}{4})$ med $-\infty < n < \infty$

$$H(0) = D_4(0) \cdot e^{-j1.5 \cdot 0} = 1 \cdot e^{j0}$$

$$H(\frac{\pi}{5}) = D_4\left(\frac{\pi}{5}\right) \cdot e^{-j1.5 \cdot \frac{\pi}{5}} = 0.769421 \cdot e^{-j\frac{3\pi}{10}}$$

$$H(\frac{\pi}{2}) = D_4\left(\frac{\pi}{2}\right) \cdot e^{-j1.5 \cdot \frac{\pi}{2}} = 0 \cdot e^{-j\frac{3\pi}{4}}$$

$$y[n] = 1 \cdot 5 + 0.769421 \cdot 4\cos(\frac{\pi}{5} n - \frac{3\pi}{10}) + 0 \cdot 3\cos(\frac{\pi}{2} n + \frac{\pi}{4} - \frac{3\pi}{4})$$

$$\underline{= 5 + 3.08\cos(\frac{\pi}{5} n - \frac{3\pi}{10})}$$

e) Det transiente forløb er forbi for $n > M$, dvs. $y[n] = y_1[n]$ for $\underline{n \geq 3}$.