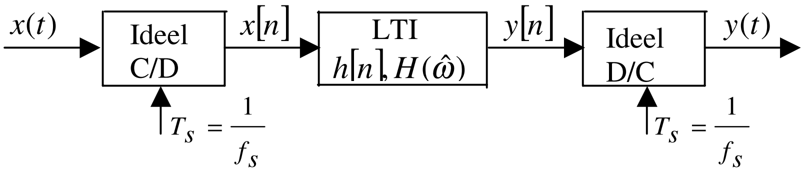
<u>Opg 6.19</u>

$$x(t) = 10 + 20\cos(\omega_0 t + \frac{\pi}{3}) \qquad -\infty < t < \infty$$



a) $h[n] = \delta[n] \qquad \omega_0 = 2\pi \cdot 500$

For hvilke værdier af f_s vil $y(t) = x(t)$?

$h[n] = \delta[n] \quad \Rightarrow \quad H(\hat{\omega}) = 1 \quad \Rightarrow \quad y[n] = x[n].$

Derfor bliver $y(t) = x(t)$, bare samplefrekvensen er høj nok, dvs.
$f_s > 2 f_0 = \underline{1000[Hz]}$

b) $h[n] = h[n-10]$. Bestem f_s og værdier af ω_0, så
$y(t) = x(t - 0.001) = 10 + 20\cos\left[\omega_0 (t - 0.001) + \frac{\pi}{3}\right]$.

$$h[n] = h[n-10] \qquad \Rightarrow \qquad H(\hat{\omega}) = 1 \cdot e^{j\hat{\omega}10}$$

$$\hat{\omega}_0 = \omega \cdot T_s \qquad t = nT_s$$

$$x[n] = 10 + 20\cos(\hat{\omega}_0 n + \frac{\pi}{3})$$

$$H(0) = 1 \qquad H(\hat{\omega}_0) = e^{-j\hat{\omega}_0 10}$$

$$\Rightarrow \qquad y[n] = 1 \cdot 10 + 1 \cdot 20\cos(\hat{\omega}_0 n + \frac{\pi}{3} - \hat{\omega}_0 10)$$

$$= 10 + 20\cos(\hat{\omega}_0 (n - 10) + \frac{\pi}{3})$$

$$= 10 + 20\cos(\omega_0 (nT_s - 10T_s) + \frac{\pi}{3})$$

$\Rightarrow \quad y(t) = 10 + 20\cos(\omega_0(t - 10T_s) + \frac{\pi}{3})$

$= 10 + 20\cos(\omega_0(t - 0.001) + \frac{\pi}{3})$

$\Leftrightarrow \quad T_s = 10^{-4}[s]$ eller $\underline{f_s = 10[kHz]}$

For korrekt sampling kræves desuden $f_s > 2f_0$ eller at

$f_0 < \frac{f_s}{2} = \frac{10^4}{2} = \underline{5000[Hz]}$

c) Glidende middelværdi. $M = 5$.

$H(\hat{\omega}) = \frac{\sin(5\hat{\omega}/2)}{5\sin(\hat{\omega}/2)} e^{-j\hat{\omega}2} = D_5(\hat{\omega})e^{-j\hat{\omega}2} \qquad f_s = 2000[s^{-1}]$

$x[n] = 10 + 20\cos(\hat{\omega}_0 n + \frac{\pi}{3})$

Hvis $y[n] = konst.$, må der gælde: $H(\hat{\omega}_0) = 0$.

$D_5(\hat{\omega}_0) = 0 \quad \Leftrightarrow \quad \sin(5\hat{\omega}_0/2) = 0 \quad \wedge \quad \sin(\hat{\omega}_0/2) \neq 0$

$\Leftrightarrow \quad \hat{\omega}_0 = \pm\frac{2\pi}{5}, \pm\frac{4\pi}{5}, \ldots$

$\omega_0 = \frac{\hat{\omega}_0}{T_s} = \hat{\omega}_0 f_s = \frac{2\pi}{5}\cdot 2000 = \underline{800\pi} = 7896[s^{-1}] \quad \Rightarrow \quad \underline{f_0 = 400[Hz]}.$

eller $\omega_0 = \frac{\hat{\omega}_0}{T_s} = \hat{\omega}_0 f_s = \frac{4\pi}{5}\cdot 2000 = \underline{1600\pi} = 15791[s^{-1}] \quad \Rightarrow \quad \underline{f_0 = 800[Hz]}.$

Bemærk, at begge frekvenser tilfredsstiller Shannons teorem.

$H(0) = D_5(0)e^{-j02} = 1 \quad \Rightarrow \quad y[n] = 1\cdot 10 + 0\cdot 20\cos(\hat{\omega}_0 n + \frac{\pi}{3}) = 10$

$\Rightarrow \quad y(t) = 10 \quad \Rightarrow \quad \underline{A = 10}.$