

Opg 6.4

$$\text{LTI: } y[n] = 2x[n] - 3x[n-1] + 2x[n-2]$$

$$\{b_k\} = \{2, -3, 2\} \quad L = 3, \quad M = 2$$

$$\text{a) } H(\hat{\omega}) = \sum_{k=0}^2 b_k e^{-j\hat{\omega}k} = \underline{2 - 3e^{-j\hat{\omega}} + 2e^{-j\hat{\omega}2}} \quad (\text{I})$$

$$= e^{-j\hat{\omega}} (2e^{j\hat{\omega}} - 3 + 2e^{-j\hat{\omega}}) = e^{-j\hat{\omega}} (4\cos \hat{\omega} - 3)$$

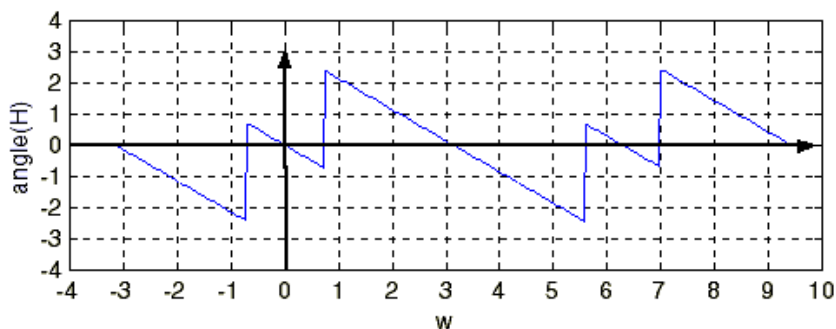
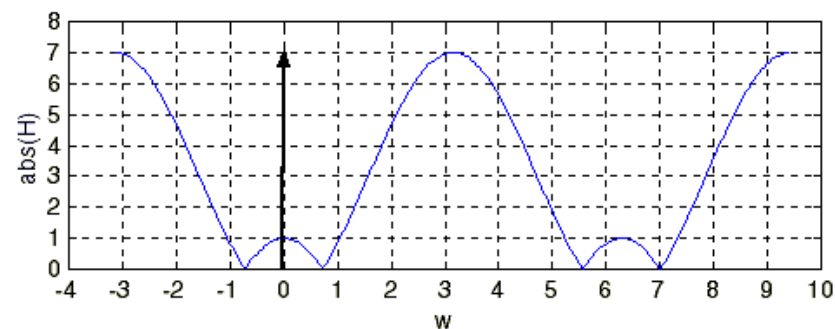
$$4\cos \hat{\omega} - 3 < 0 \quad \Leftrightarrow \quad \cos \hat{\omega} < \frac{3}{4}$$

$$\Leftrightarrow \quad -\pi < \hat{\omega} < -0.723 \quad \text{og} \quad 0.723 < \hat{\omega} < \pi$$

$$H(\hat{\omega}) = \begin{cases} |4\cos \hat{\omega} - 3| e^{-j(\hat{\omega}+\pi)} & -\pi < \hat{\omega} < -0.723 \\ |4\cos \hat{\omega} - 3| e^{-j\hat{\omega}} & -0.723 < \hat{\omega} < 0.723 \\ |4\cos \hat{\omega} - 3| e^{-j(\hat{\omega}-\pi)} & 0.723 < \hat{\omega} < \pi \end{cases}$$

- b) (I) er en linear kombination af harmoniske.
Laveste frekvens er $\hat{\omega}$, dvs. periodicitet: $\underline{2\pi}$.

c)



Mathlab-kode, der genererer figurene:

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» w=-pi:0.05:3*pi;
» B=[2 -3 2];
» A=[1];
» H=freqz(B,A,w);
» subplot(2,1,1), plot(w,abs(H))
» subplot(2,1,2), plot(w,angle(H))
»

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Figurene kan også fås med TI-89 eller TI-92. Faseplottet giver netop hovedværdien, som også vist i ovenstående Mathlab-plot.

$$d) \quad x[n] = e^{j\hat{\omega}n}$$

$$y[n] = H(\hat{\omega}) \cdot x[n] = e^{-j\hat{\omega}n} (4 \cos \hat{\omega} - 3) \cdot e^{j\hat{\omega}n}$$

$$y[n] = 0 \quad \Leftrightarrow \quad 4 \cos \hat{\omega} - 3 = 0$$

$$\Leftrightarrow \quad \hat{\omega} = \begin{cases} 0.723 + p \cdot 2\pi \\ -0.723 + p \cdot 2\pi \end{cases} \quad p \in Z$$

$$e) \quad x[n] = \sin\left(\frac{\pi n}{13}\right) = \cos\left(\frac{\pi}{2} - \frac{\pi n}{13}\right) = \cos\left(\frac{\pi n}{13} - \frac{\pi}{2}\right)$$

$$y[n] = |H(\hat{\omega})| A \cos(\hat{\omega}n + \phi + \arg H(\hat{\omega})) \quad \text{se evt. p. 161}$$

$$\hat{\omega} = \frac{\pi}{13} = 0.242, \text{ dvs. } 0 < \hat{\omega} < 0.723, \text{ dvs. } \arg H(\hat{\omega}) = -\hat{\omega} = 0.242 = -\frac{\pi}{13}.$$

$$\text{Her er } A = 1 \text{ og } \phi = -\frac{\pi}{2}, \text{ hvoraf } H\left(\frac{\pi}{13}\right) = |4 \cos 0.242 - 3| = 0.884$$

$$y[n] = 0.884 \cdot 1 \cos\left(\frac{\pi}{13}n - \frac{\pi}{2} - \frac{\pi}{13}\right) = 0.884 \cos\left(\frac{\pi}{13}(n-1) - \frac{\pi}{2}\right)$$

$$= \underline{\underline{0.884 \sin\left(\frac{\pi}{13}(n-1)\right)}}$$