

Opg 6.6

$$\text{LTI: } y[n] = x[n] - x[n-2]$$

$$\{b_k\} = \{1, 0, -1\} \quad L = 3, \quad M = 2$$

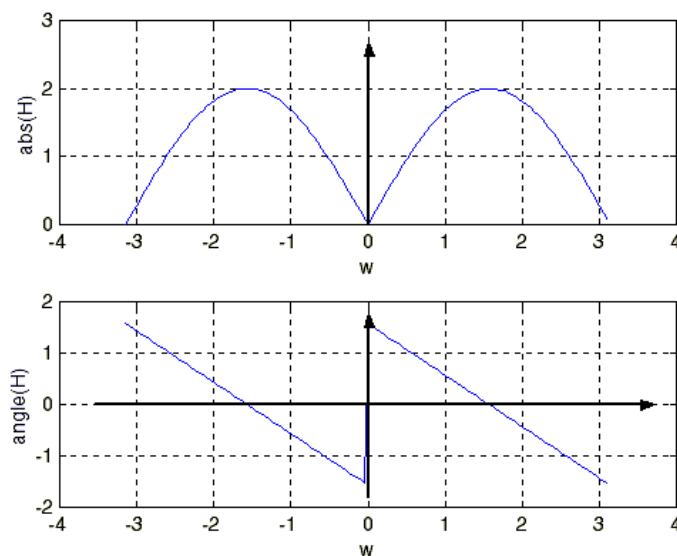
$$\text{a) } H(\hat{\omega}) = \sum_{k=0}^2 b_k e^{-j\hat{\omega}k} = \underline{1 - e^{-j\hat{\omega}2}}$$

$$= e^{-j\hat{\omega}} (e^{j\hat{\omega}} - e^{-j\hat{\omega}}) = 2je^{-j\hat{\omega}} \sin \hat{\omega} = \underline{2 \sin \hat{\omega} \cdot e^{-j(\hat{\omega} - \frac{\pi}{2})}}$$

b) $\sin \hat{\omega} < 0$ for $-\pi < \hat{\omega} < 0$, altså:

$$H(\hat{\omega}) = \begin{cases} 2|\sin \hat{\omega}| \cdot e^{-j(\hat{\omega} + \frac{\pi}{2})} & -\pi < \hat{\omega} < 0 \\ 2|\sin \hat{\omega}| \cdot e^{-j(\hat{\omega} - \frac{\pi}{2})} & 0 < \hat{\omega} < \pi \end{cases}$$

$$|H(\hat{\omega})| = 2|\sin \hat{\omega}| \quad \arg H(\hat{\omega}) = \begin{cases} -(\hat{\omega} + \frac{\pi}{2}) & -\pi < \hat{\omega} < 0 \\ -(\hat{\omega} - \frac{\pi}{2}) & 0 < \hat{\omega} < \pi \end{cases}$$



Mathlab-kode, der genererer figurerne:

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» w=-pi:0.05:pi;
» B=[1 0 -1];
» A=[1];
» H=freqz(B,A,w);
» subplot(2,1,1), plot(w,abs(H))
» subplot(2,1,2), plot(w,angle(H))
»

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c) Input: $x[n] = 4 + \cos(0.25\pi n - \frac{\pi}{4})$. Bestem output $y[n]$.

$$\hat{\omega} = 0: \quad |H(0)| = 2|\sin 0| = 0 \quad \arg H(0) = \pm \frac{\pi}{2}$$

$$\hat{\omega} = \frac{\pi}{4}: \quad \left| H\left(\frac{\pi}{4}\right) \right| = 2 \left| \sin \frac{\pi}{4} \right| = \sqrt{2} \quad \arg H\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$$

$$y[n] = 0 \cdot 4 + \sqrt{2} \cdot \cos\left(\frac{\pi}{4}n - \frac{\pi}{4} + \frac{\pi}{4}\right) = \sqrt{2} \cos\left(\frac{\pi}{4}n\right)$$

d) Input: $x_1[n] = \left(4 + \cos(0.25\pi n - \frac{\pi}{4}) \right) u[n]$.

Den transiente del er forbi for: $n \geq M = 2$.