

Opg 6.7

LTI-system

$$\begin{aligned}
 \text{a)} \quad H(\hat{\omega}) &= (1 + e^{-j\hat{\omega}})(1 - e^{j\frac{2\pi}{3}} e^{-j\hat{\omega}})(1 - e^{-j\frac{2\pi}{3}} e^{-j\hat{\omega}}) \\
 &= (1 + e^{-j\hat{\omega}})(1 - e^{-j\frac{2\pi}{3}} e^{-j\hat{\omega}} - e^{j\frac{2\pi}{3}} e^{-j\hat{\omega}} + e^{j\frac{2\pi}{3}} e^{-j\frac{2\pi}{3}} e^{-j2\hat{\omega}}) \\
 &= (1 + e^{-j\hat{\omega}})(1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}) = 1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}} \\
 &= 1 + 2e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}} \quad \{b_k\} = \{1, 2, 2, 1\}
 \end{aligned}$$

$$\Rightarrow \underline{y[n] = x[n] + 2x[n-1] + 2x[n-2] + x[n-3]}$$

$$\text{b)} \quad \text{Input: } x[n] = \delta[n] \Rightarrow$$

$$\text{Output: } \underline{h[n] = \delta[n] + 2\delta[n-1] + 2\delta[n-2] + \delta[n-3]}$$

$$\text{c)} \quad \text{Input: } x[n] = Ae^{j\phi} e^{j\hat{\omega}n}$$

Hvilke værdier af $\hat{\omega}$, hvor $-\pi \leq \hat{\omega} \leq \pi$, vil bevirke, at outputtet $y[n]$ bliver nul for alle værdier af n ?

$$y[n] = H(\hat{\omega}) Ae^{j\phi} e^{j\hat{\omega}n} = 0 \text{ for alle } n \Leftrightarrow H(\hat{\omega}) = 0 \Leftrightarrow$$

en af faktorerne i $H(\hat{\omega})$ er nul. Dvs.

$$1 + e^{-j\hat{\omega}} = 0 \quad \vee \quad 1 - e^{j\frac{2\pi}{3}} e^{-j\hat{\omega}} = 0 \quad \vee \quad 1 - e^{-j\frac{2\pi}{3}} e^{-j\hat{\omega}} = 0$$

$$\Leftrightarrow \underline{\hat{\omega} = \pm\pi} \quad \vee \quad \underline{\hat{\omega} = \frac{2\pi}{3}} \quad \vee \quad \underline{\hat{\omega} = -\frac{2\pi}{3}}$$