

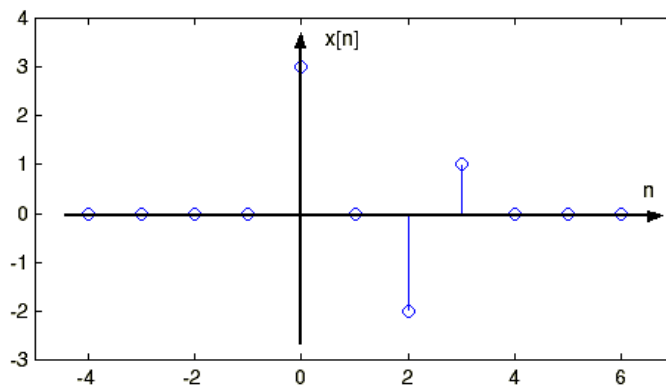
Opg 6.9

LTI-system Tre eksperimenter:

1) $x[n] = \delta[n] \Rightarrow y[n] = \delta[n] - \delta[n-3]$

2) $x[n] = \cos\left(\frac{2\pi}{3}n\right) \Rightarrow y[n] = 0$

3) $x[n] = \cos\left(\frac{\pi}{3}n + \frac{\pi}{2}\right) \Rightarrow y[n] = 2\cos\left(\frac{\pi}{3}n + \frac{\pi}{2}\right)$

a) Plot signalet: $x[n] = 3\delta[n] - 2\delta[n-2] + \delta[n-3]$ b) Hvad bliver output med ovennævnte inputsignal? Output fra første test er impulssvaret $h[n]$. Vi har derfor $y[n] = x[n] * h[n]$.

| | | | | | | | |
|------------|---|---|----|----|---|---|----|
| n | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| x[n] | 3 | 0 | -2 | 1 | 0 | 0 | 0 |
| h[n] | 1 | 0 | 0 | -1 | 0 | 0 | 0 |
| h[0]x[n-0] | 3 | 0 | -2 | 1 | 0 | 0 | 0 |
| h[1]x[n-1] | | 0 | 0 | 0 | 0 | 0 | 0 |
| h[2]x[n-2] | | | 0 | 0 | 0 | 0 | 0 |
| h[3]x[n-3] | | | | -3 | 0 | 2 | -1 |
| y[n] | 3 | 0 | -2 | -2 | 0 | 2 | -1 |

Mathlab: $h=[1\ 0\ 0\ -1];$
 $x=[3\ 0\ -2\ 1];$
 $y=conv(h,x)$

Man kan i stedet udnytte, at systemet er lineært og tidsinvariant, sammen med resultatet af første eksperiment:

$$\begin{aligned} y[n] &= 3(\delta[n] - \delta[n-3]) - 2(\delta[(n-2)-0] - \delta[(n-2)-3]) \\ &\quad + (\delta[(n-3)-0] - \delta[(n-3)-3]) \\ &= \underline{3\delta[n] - 2\delta[n-2] - 2\delta[n-3] + 2\delta[n-5] - \delta[n-6]} \end{aligned}$$

c) For sinusformedede signaler gælder: $y[n] = H(\hat{\omega})x[n]$

Test nr. 3 giver derfor: $H\left(\frac{\pi}{3}\right) = 2e^{j0} = 2.$

Hermed fås også:

$$x[n] = \cos\left(\frac{\pi}{3}n - \pi\right) \Rightarrow \underline{y[n] = 2\cos\left(\frac{\pi}{3}n - \pi\right)}$$

d) $\{b_k\} = \{1, 0, 0, -1\} \Rightarrow H(\hat{\omega}) = 1 - e^{-j3\hat{\omega}}$

$$\Rightarrow H\left(\frac{\pi}{2}\right) = 1 - e^{-j\frac{3\pi}{2}} = \underline{1 - j \neq 0}$$

Bonus:

$$\begin{aligned} H(\hat{\omega}) &= 1 - e^{-j3\hat{\omega}} = e^{-j1.5\hat{\omega}} (e^{j1.5\hat{\omega}} - e^{-j1.5\hat{\omega}}) = 2je^{-j1.5\hat{\omega}} \sin(1.5\hat{\omega}) \\ &= 2\sin(1.5\hat{\omega})e^{j\left(\frac{\pi}{2} - 1.5\hat{\omega}\right)} \end{aligned}$$

$$\Rightarrow |H(\hat{\omega})| = 2|\sin(1.5\hat{\omega})|$$

$$\text{og } \arg H(\hat{\omega}) = \begin{cases} -\frac{3\pi}{2} - 1.5\hat{\omega} & -\pi < \hat{\omega} < -\frac{2\pi}{3} \\ -\frac{\pi}{2} - 1.5\hat{\omega} & -\frac{2\pi}{3} < \hat{\omega} < 0 \\ \frac{\pi}{2} - 1.5\hat{\omega} & 0 < \hat{\omega} < \frac{2\pi}{3} \\ \frac{3\pi}{2} - 1.5\hat{\omega} & \frac{2\pi}{3} < \hat{\omega} < \pi \end{cases}$$

Mathlab-kode, der genererer figurene:

```
» w=-pi:0.05:pi;  
» B=[1 0 -1];  
» A=[1];  
» H=freqz(B,A,w);  
» plot(w,abs(H))  
» plot(w,angle(H))  
»
```

