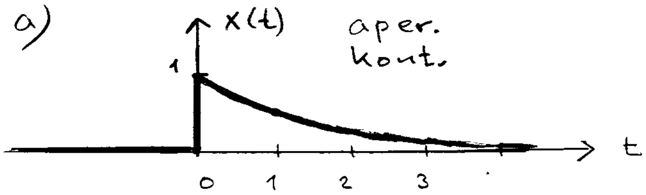
$x(t) = a^t u(t)$ og $x[n] = \begin{cases} a^n & 0 \le n \le N-1 \\ 0 & \text{ellers} \end{cases}$ $\quad a = 1/2,\ N = 4$

a)



t	0	1	2	3
x(t)	1	1/2	1/4	1/8

b) $\underline{X}(j\Omega) \equiv \int_{-\infty}^{\infty} x(t) e^{-j\Omega t}\, dt = \int_0^{\infty} a^t e^{-j\Omega t}\, dt$

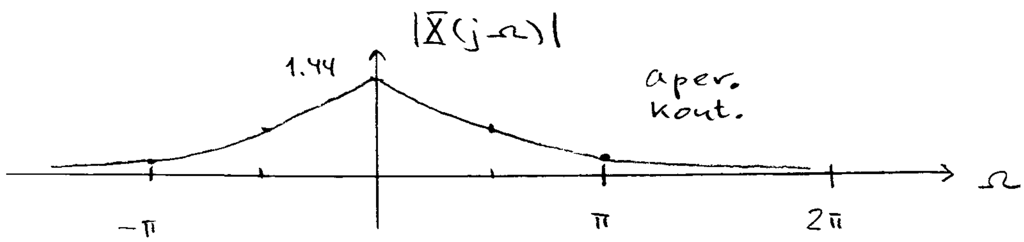
$$= \int_0^{\infty} \left(e^{\ln a \cdot t}\right) \cdot e^{-j\Omega t}\, dt = \int_0^{\infty} e^{(\ln a - j\Omega)t}\, dt = \left[\frac{e^{(\ln a - j\Omega)t}}{\ln a - j\Omega}\right]_0^{\infty}$$

$$= \left[\frac{a^t \cdot e^{-j\Omega t}}{\ln a - j\Omega}\right]_0^{\infty} \longrightarrow 0 - \frac{a^0 e^{-j\Omega 0}}{\ln a - j\Omega} = \underline{\underline{\frac{1}{j\Omega - \ln a}}}$$

$$= |\underline{X}(j\Omega)| \cdot e^{j \arg \underline{X}(j\Omega)}$$

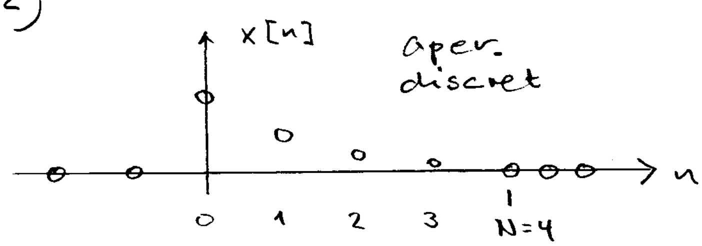
$$|\underline{X}(j\Omega)| = \left|\frac{1}{j\Omega - \ln a}\right| = \underline{\underline{\frac{1}{\sqrt{(-\ln a)^2 + \Omega^2}}}} = \frac{1}{\sqrt{0.480 + \Omega^2}}$$

Ω	$-\pi$	$-\pi/2$	0	$\pi/2$	π
$\lvert\underline{X}(j\Omega)\rvert$	0.311	0.582	1.44	0.582	0.311



Bemærk, at x(t) ikke er båndbegrænset, men store frekvenser har små amplituder.
Alligevel kan de høje frekvenser godt indeholde meget energi (der er jo uendelig mange af dem).

c)



n	0	1	2	3
x[n]	1	1/2	1/4	1/8

d) $\bar{X}(e^{j\omega}) \equiv \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-j\omega n} = \sum_{n=0}^{3} x[n] \cdot e^{-j\omega n}$

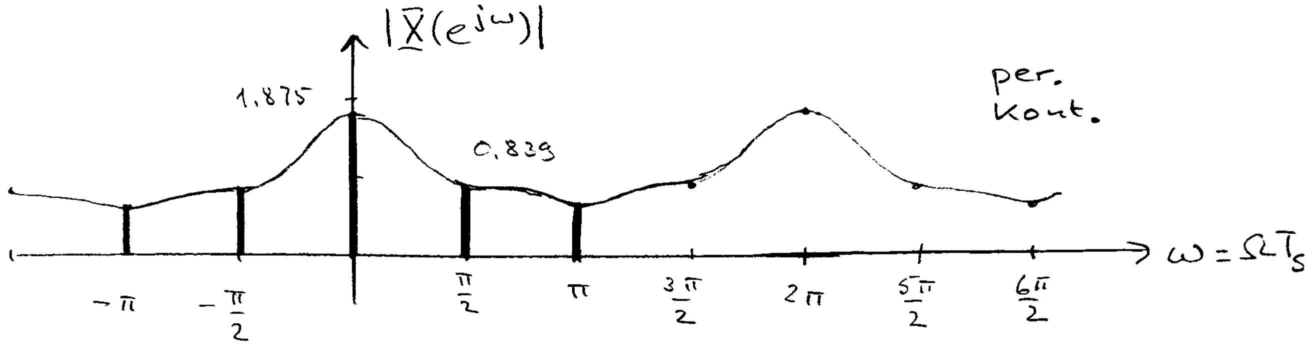
$$= \sum_{n=0}^{3} a^n \cdot e^{-j\omega n} = \sum_{n=0}^{3} (a e^{-j\omega})^n = \underline{\underline{\frac{1 - a^4 e^{-j\omega 4}}{1 - a e^{-j\omega}}}}$$

$$= \underline{\underline{1 + a e^{-j\omega} + a^2 e^{-j2\omega} + a^3 e^{-j3\omega}}}$$

Der gælder: $\omega \equiv \Omega T_s = \Omega \cdot 1 = \Omega \quad (T_s = 1)$

$a = \frac{1}{2}$

ω	$-\pi$	$-\pi/2$	0	$\pi/2$	π
$\lvert\bar{X}(e^{j\omega})\rvert$	$\frac{10}{16}$	$\frac{30\sqrt{5}}{16}$	$\frac{30}{16}$	$\frac{30/\sqrt{5}}{16}$	$\frac{10}{16}$
	0,625	0,839	1,875	0,839	0,625

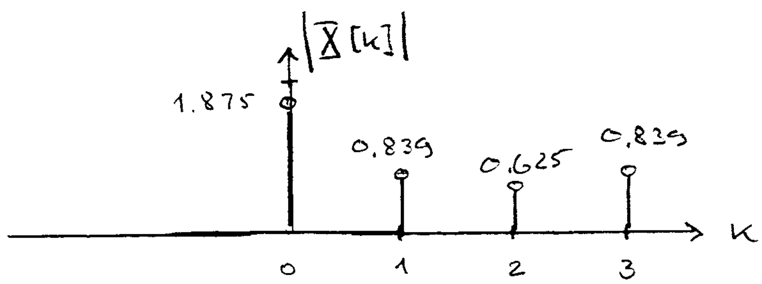


Skitseret ved hjælp af TI-89.

e) $\bar{X}[k] \equiv \sum_{n=0}^{N-1} x[n] \cdot W_N^{kn} = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn} \qquad (8.67)$

$$= \sum_{n=0}^{3} a^n \cdot e^{-j\frac{2\pi}{4} \cdot k \cdot n} = \underline{\underline{1 + a e^{-j\frac{\pi}{2}k} + a^2 e^{-j\pi k} + a^3 e^{-j\frac{3\pi}{2} \cdot k}}}$$

k	0	1	2	3
$\lvert\bar{X}[k]\rvert$	1,875	0,839	0.625	0.839



Det ses, at $\lvert\bar{X}[k]\rvert$ er discrete samples af $\lvert X(e^{j\omega})\rvert$.

Der gælder: $\omega = \frac{2\pi}{N} \cdot k$. $k = 3$ svarer til $\omega = -\frac{\pi}{2}$.

Af tabellerne ses, at $\lvert X[k]\rvert$ kun er en tilnærmelse til sampes af $\lvert\bar{X}(j\Omega)\rvert$. NB! $x(t)$ er ikke båndbegrænset.